**FEDERAL PUBLIC SERVICE COMMISSION**
**COMPETITIVE EXAMINATION-2017**
**FOR RECRUITMENT TO POSTS IN BS-17**
**UNDER THE FEDERAL GOVERNMENT**

| **Roll Number** |
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# PURE MATHEMATICS

| **TIME ALLOWED: THREE HOURS** | **MAXIMUM MARKS = 100** |
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**NOTE: (i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.

**(ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.

**(iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.

**(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.

**(v)** Extra attempt of any question or any part of the attempted question will not be considered.

**(vi)** **Use of Calculator is allowed.**

## SECTION-A

**Q. 1.** **(a)** Let $H$, $K$ be subgroups of a group $G$. Prove that $HK$ is a subgroup of $G$ if and only if $HK=KH$. (10)

**(b)** If $N$, $M$ are normal subgroups of a group $G$, prove that (10) **(20)**

$$NM/M \cong N/N\cap M.$$

**Q. 2.** **(a)** If $R$ is a commutative ring with unit element and $M$ is an ideal of $R$ then show that $M$ is a maximal ideal of $R$ if and only if $R/M$ is a field. (10)

**(b)** If $F$ is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of $F$ then show that we can find elements $a$ and $b$ in $F$ such that (10) **(20)**

$$1 + \alpha a^2 + \beta b^2 = 0.$$

**Q. 3.** **(a)** Let $V$ be a finite-dimensional vector space over a field $F$ and $W$ be a subspace of $V$. Then show that $W$ is finite-dimensional, (10)

$$\dim W \leq \dim V \text{ and } \dim V/W = \dim V - \dim W.$$

**(b)** Suppose $V$ is a finite-dimensional vector space over a field $F$. Prove that a linear transformation $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for $T$ is not 0. (10) **(20)**

## SECTION-B

**Q. 4.** **(a)** Use the Mean-Value Theorem to show that if $f$ is differentiable on an interval $I$, and if $|f'(x)| \leq M$ for all values of $x$ in $I$, then (10)

$$|f(x) - f(y)| \leq M|x - y|$$

for all values of $x$ and $y$ in $I$. Use this result to show further that

$$|\sin x - \sin y| \leq |x - y|.$$

**(b)** Prove that if $x = x(t)$ and $y = y(t)$ are differentiable at $t$, and if $z = f(x, y)$ is differentiable at the point $(x, y) = \big(x(t), y(t)\big)$, then $z = f\big(x(t), y(t)\big)$ is differentiable at $t$ and (10) **(20)**

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

where the ordinary derivatives are evaluated at $t$ and the partial derivatives are evaluated at $(x, y)$.

**Q. 5.** **(a)** Evaluate the double integral (10)

$$\iint_R (3x - 2y)\, dx\, dy$$

**(b)** Where $R$ is a region enclosed by the circle $x^2 + y^2 = 1$. (10) **(20)**

Find the area of the region enclosed by the curves

$$y = \sin x, \quad y = \cos x, x = 0, \quad x = 2\pi.$$

**Q. 6.** **(a)** Find an equation of the ellipse traced by a point that moves so that the sum of its distance to (4,1) and (4,5) is 12. (10)

**(b)** Show that if $a$, $b$ and $c$ are nonzero, then the plane whose intercepts with the coordinate axes are $x = a$, $y = b$, and $z = c$ is given by the equation. (10) **(20)**

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

## SECTION-C

**Q. 7.** **(a)** Prove that a necessary and sufficient condition that (10)

$$w = f(z) = u(x, y) + iv(x, y)$$

be analytic in a region $R$ is that the Cauchy-Riemann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{and} \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

are satisfied in $R$ where it is supposed that these partial derivatives are continuous in $R$.

**(b)** Show that the function $f(z) = \bar{z}$ is not analytic anywhere in the complex plane $Z$. (10) **(20)**

**Q. 8.** **(a)** Let $f(z)$ be analytic inside and on the boundary $C$ of a simply-connected region $R$. Prove that (10)

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz.$$

**(b)** Show that

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2} = \frac{5\pi}{32}.$$ (10) **(20)**

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